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LETTER TO THE EDITOR

Deterministic self-avoiding walker in a random medium

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Abstract. A variant of 'true self-avoiding walk' (TSAW) is studied. The walker chooses the most favourable direction as the next step in a lattice with random bonds. The process is deterministic and our numerical simulation in one dimension indicates that the scaling behaviour is the same as TSAW, i.e. $x \sim t^{2/3}$, though there is no apparent mapping between the two models.

In many kinetic processes there are two kinds of randomness: one is that the process evolves in time via probability, e.g. it is intrinsically stochastic; examples are the Eden model, diffusion-limited aggregation (DLA) (Witten and Sander 1981) and true self-avoiding walk (TSAW) (Amit *et al* 1983); the other is due to the fact that the process takes place in random media and can be described deterministically. For example, the invasion percolation (IP) (Wilkinson and Willemson 1983). Obviously many problems can have both types of randomness.

We would like to know what effects these two types of randomness have on the process separately, and the relationship between them. Recently it has been found that the DLA cluster can be obtained deterministically in a random medium (Chen and Wilkinson 1985), while it is known that the invasion percolation cluster is very different from the stochastic one (Wilkinson and Willemson 1983).

In this letter we propose a simple model where these ideas can be confronted. We take a lattice where, associated with each bond, there is an independent quenched random number. We consider a walker on this lattice sitting on a certain site. Next time it will go to a nearest-neighbour site by choosing the maximal value of $\exp(-gn+b)$, where n is the number of previous visits at one of the nearest-neighbour sites, g is a constant and b is the Gaussian quenched randomness. We see that this model is just a variant of TSAW; the differences are that the lattice has quenched randomness and the process is deterministic.

We studied the model by numerical simulations. We consider here only the one-dimensional case. The counterpart of TSAW is known to have a non-trivial scaling relation in one dimension as $x \sim t^{2/3}$, both numerically (Bernasconi and Pietronero 1983) and by a scaling argument (Pietronero 1983). We took a space chain of 3000 sites and simulated walkers up to 2^{19} time steps with up to 500 samples to average. We find that the data scale quite well (after 2^{10} steps) and give a scaling relation $x \sim t^{\alpha}$, $\alpha = 0.65 \pm 0.01$ (with $g = 0.2$, 0.4, 0.6), consistent with the value $\alpha = \frac{2}{3}$, which is the TSAW value.

While for TSAW a scaling argument can be coined, a convincing argument for this model eludes us. Apparently there is no mapping between the two models; one is a quenched zero temperature process and the other is purely stochastic. There is no

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reason why *a priori* the two models have the same scaling relation in one dimension. Unfortunately we can offer no explanation of the fact that numerical findings indicate that the two models scale in the same way in one dimension.

In principle one can also perform simulations in two dimensions, which is the upper critical dimension for TSAW. However the numerical precision would be too compromised to give an accurate answer, if there is a logarithmic correction as in TSAW.

Evidently a scaling theory is needed to provide insight into the model and field theoretical approaches may be helfpul.

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